Hints for exercises for Tuesday, second hour

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Source coding (Cover and Thomas, Exercise 3.7)

1. Count the ways you can insert black pixels into the sequence and control for the fact that the black pixels can be permuted. The exact answer is 166,751.
2. $\log_2$
3. There is a number of things to be said about this, but one important remark is that the size of the table grows exponentially with the sequence length.
4. The Markov bound gives $1/6 \approx 0.1667$. Chebyshev's inequality gives

$$\frac{100 \cdot 0.005 \cdot 0.995}{2.5^2} \approx 0.0796$$

   The actual probability is about 0.0017.

Probability threshold sets (Cover and Thomas, Exercise 3.5)

1. Probabilities must sum up to 1.
2. Start with the central limit theorem.

Random volumes (Cover and Thomas, Exercise 3.5)

1. What is $\mathsf{E}[XY]$ when X and Y are independent?
2. The first one is exponentally decreasing, but the second one requires you to know that

$$\left(1 + \frac{1}{n}\right)^n \longrightarrow e \approx 2.718.$$

3. Arguments can be given any way you like, but make sure you have something intelligent to say about the "concrete" case $n = 2$.