Exercises for Tuesday, first hour

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Chebyshev's ineqality Suppose X is random variable with expected value $\mathsf{E}[X] = \mu$ and variance $\mathsf{VAR}[X] = \sigma^2$. Prove that

$$\Pr\left(|X - \mu| \geq \varepsilon\right) \leq \frac{\sigma^2}{\varepsilon^2}.$$

This theorem is called Chebyshev's inequality.

Confidence interval Suppose a random variable X has an expected value of $\mathsf{E}[X] = \mu \geq 0$, and that most of its probability mass is located in the interval

$$[\,\mu - \sqrt{\mu},\, \mu + \sqrt{\mu}\,].$$

For which values of μ does the observation $X = 30$ lie outside this interval?

Frequentist inference A coin with an unknown bias p is flipped 100 times, and it comes up heads $S_{100} = 65$ times.

1. Suppose that $p = 0.5$. Compute the variance of each individual coin flip and the variance of S_{100}.

2. Still Assuming that $p = 0.5$, find an upper bound on the likelihood

$$\Pr\left(\,(S_{100} - 50)^2 \geq (65 - 50)^2 \,\middle|\, p = 0.5\right).$$

3. For which p does S_{100} have the greatest variance? What is the value of $\mathsf{VAR}[S_{100}]$ in this "worst" case scenario?

4. Find a reasonable upper bound on the likelihood

$$\Pr\left(\,(S_{100} - \mu)^2 \geq (65 - \mu)^2 \,\middle|\, p\right)$$

for an arbitrary p. ($\mu = \mu(p)$ is the expected value of S_{100}.)

5. Exhibit a set of p for which

$$\Pr\left(\,(S_{100} - \mu)^2 \geq (65 - \mu)^2 \,\middle|\, p\right) \leq 0.05.$$