ILLC Project Course in Information Theory

Crash course

13 January – 17 January 2014
12:00 to 14:00

Student presentations

27 January – 31 January 2014
12:00 to 14:00

Location

ILLC, room F1.15,
Science Park 107, Amsterdam

Materials

informationtheory.weebly.com

Contact

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mathias.winther@gmail.com

Monday

Probability theory
Uncertainty and coding

Tuesday

The weak law of large numbers
The source coding theorem

Wednesday

Random processes
Arithmetic coding

Thursday

Divergence
Kelly Gambling

Friday

Kolmogorov Complexity
The limits of statistics

Horse race

Horse number	1	2	3	4
Win probability	.4	.3	.2	.1
Odds	2	3	5	10

Payoffs

		Winner			
		1	2	3	4
Bet	1	$2b$	0	0	0
	2	0	$3b$	0	0
	3	0	0	$5b$	0
	4	0	0	0	$10b$

Horse race

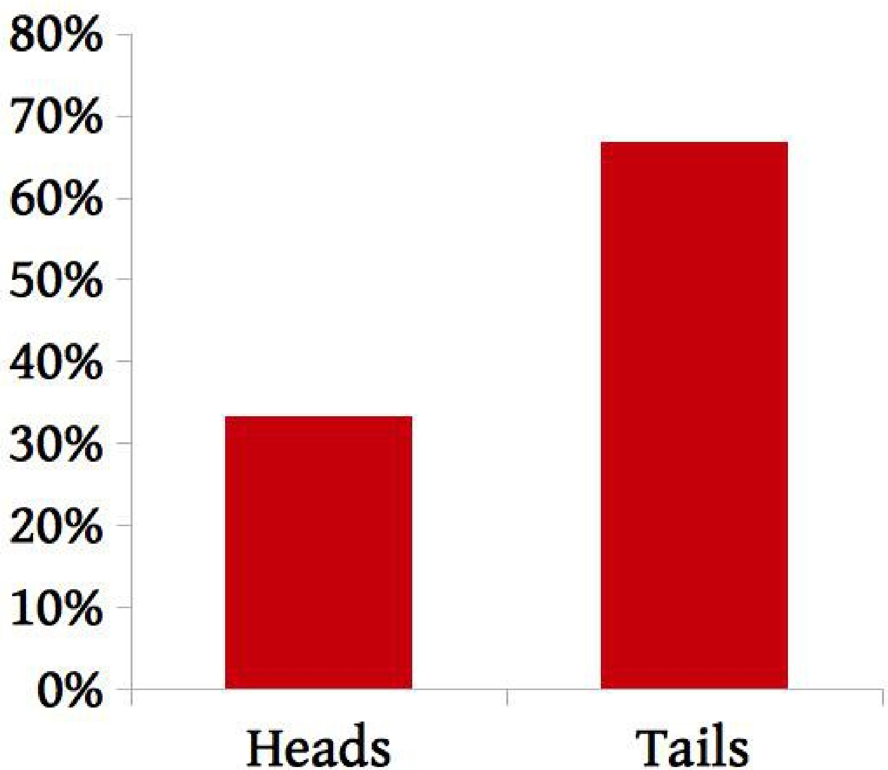
Horse number	1	2	3	4
Win probability	.4	.3	.2	.1
Odds	2	3	5	10

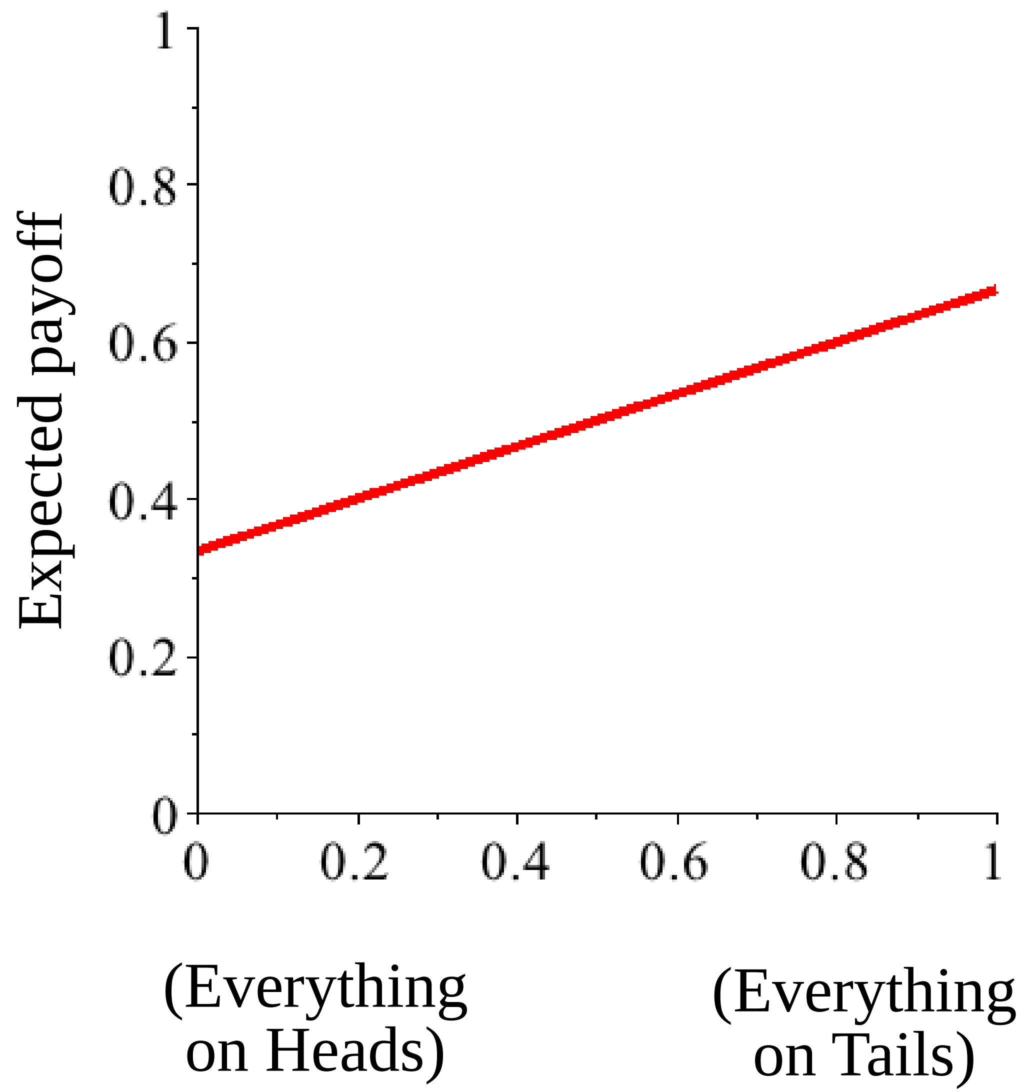
Payoffs

$$R(b) == .4 \cdot 2b + .3 \cdot 3b + .2 \cdot 5b + .1 \cdot 10b$$

$$== p_1 o_1 b_1 + p_2 o_2 b_2 + \ldots + p_n o_n b_n$$

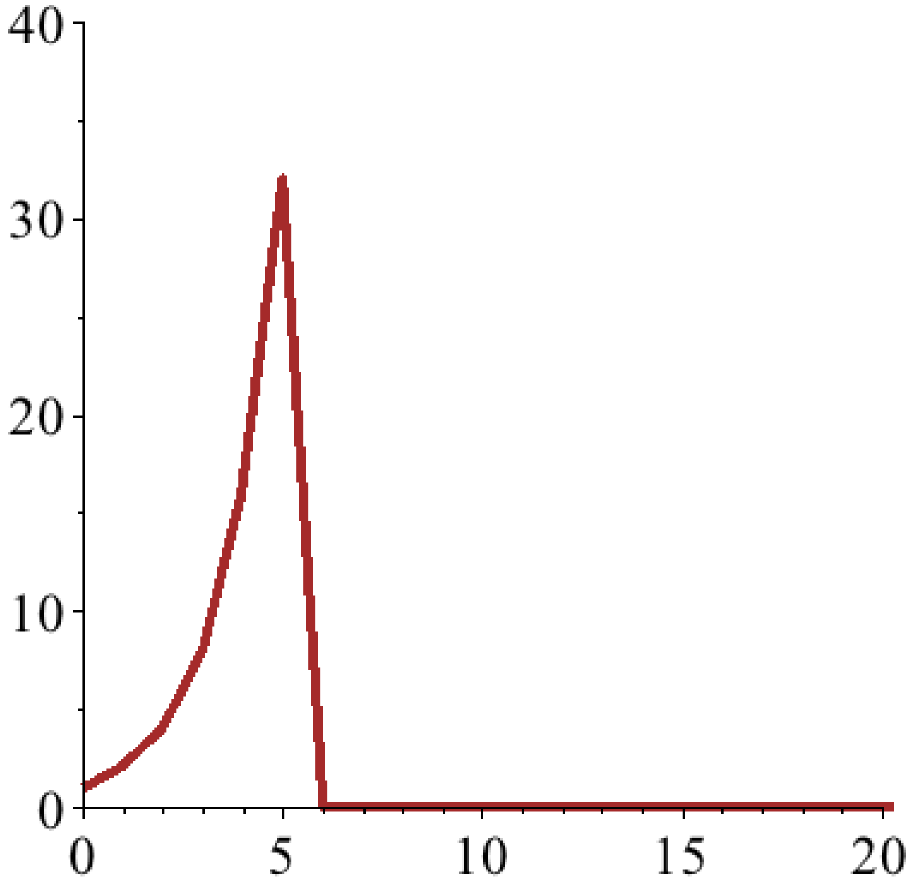
Degenerate Gambling





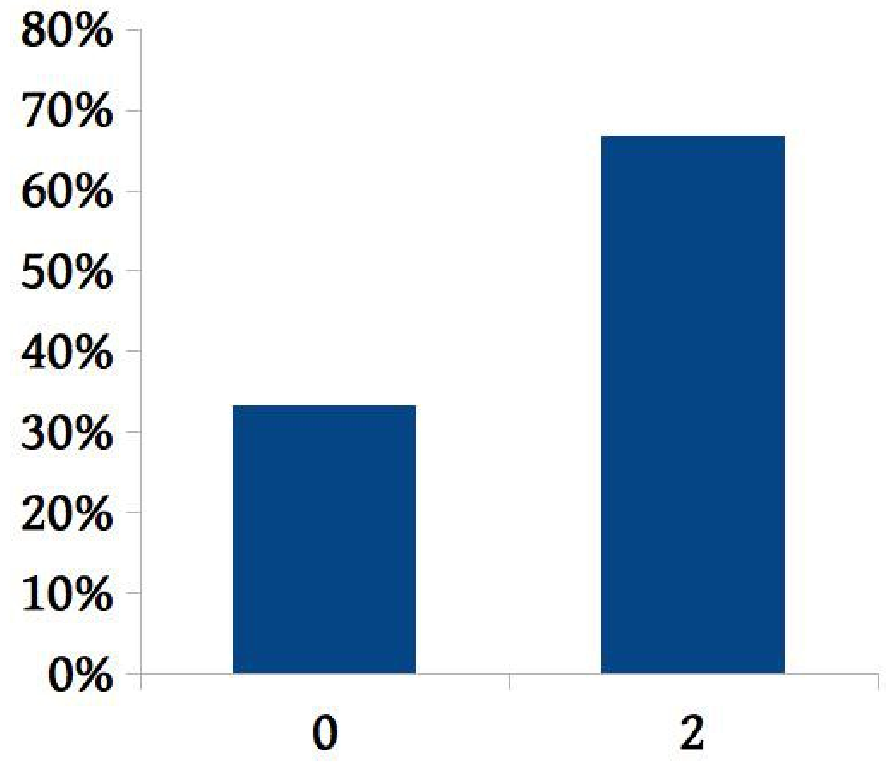
Degenerate Gambling

Capital



Rounds

Pr



Rate of return (*R*)

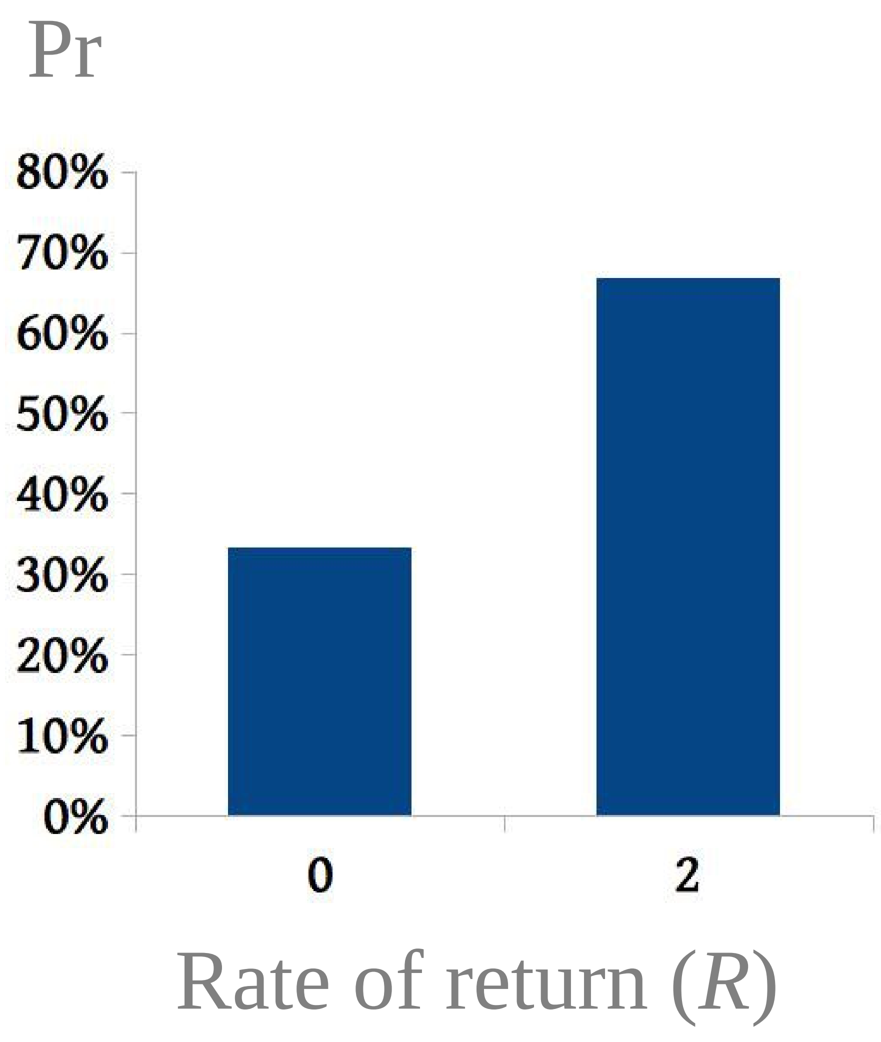
Degenerate Gambling

Rate of return:

$$R_i == \frac{\text{Capital at time } i+1}{\text{Capital at time } i}$$

Long-run behavior:

$$E[\, R_1 \cdot R_2 \cdot R_3 \cdot \cdot \cdot R_n \,]$$



Degenerate Gambling

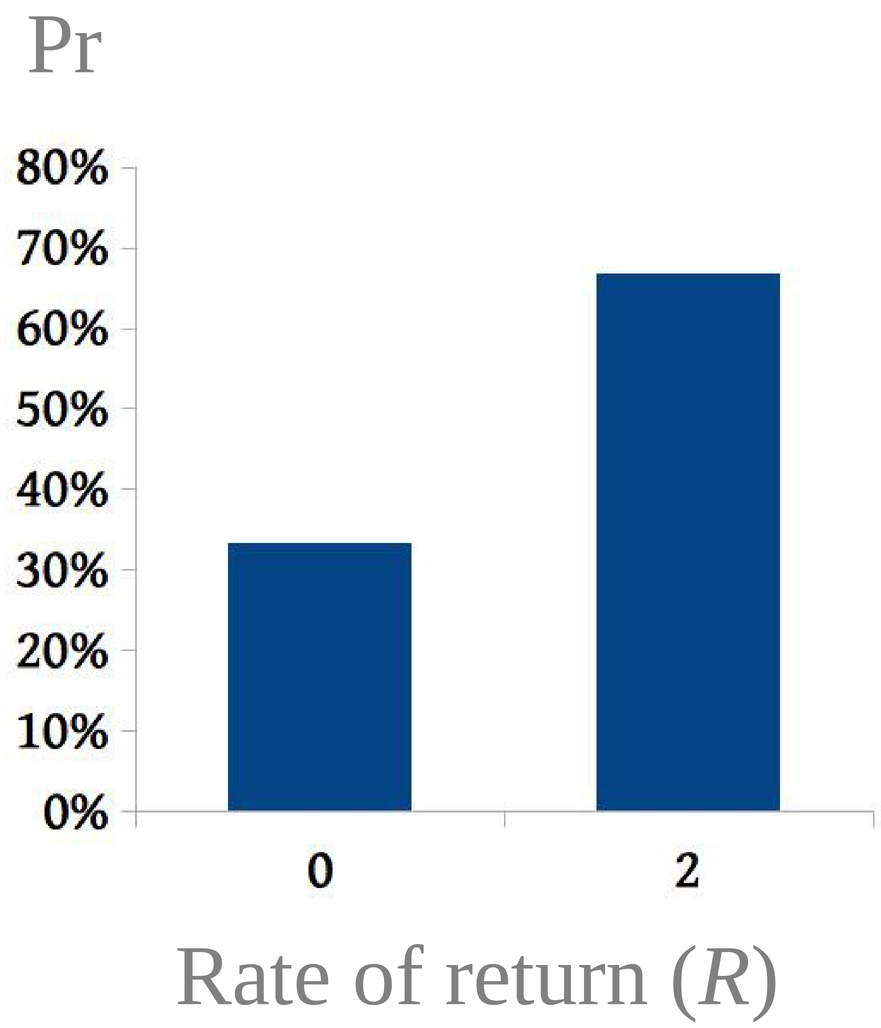
Rate of return:

$$R_i == \frac{\text{Capital at time } i+1}{\text{Capital at time } i}$$

Long-run behavior:

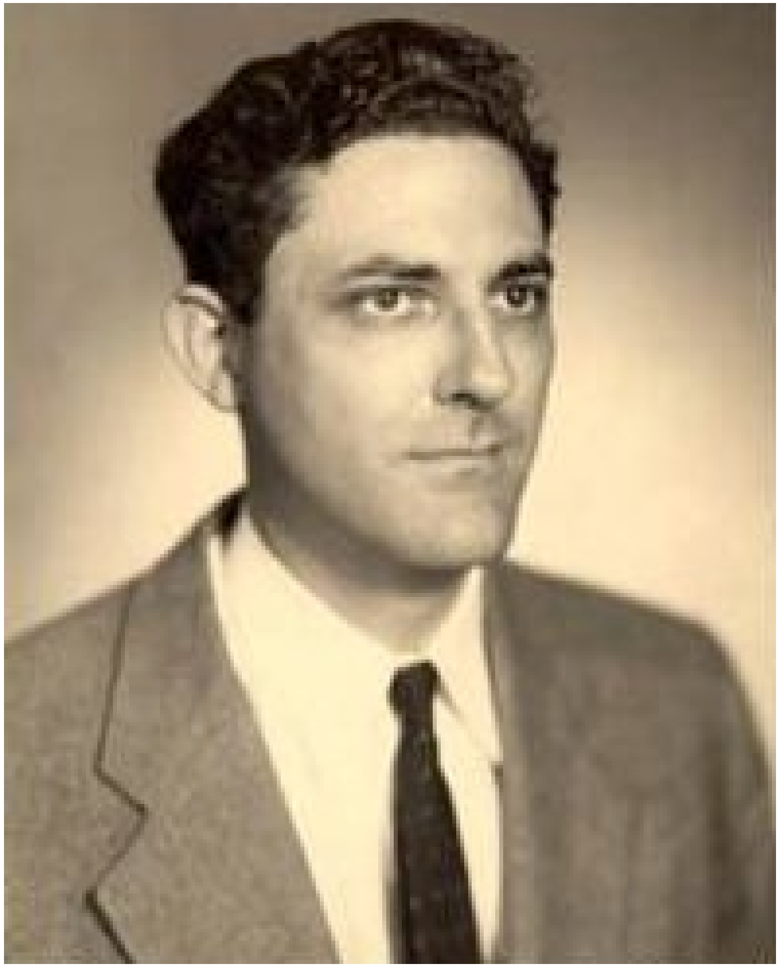
$$E[\, R_1 \cdot R_2 \cdot R_3 \cdots R_n \,]$$

0 with probability 1



Optimal reinvestment

Daniel Bernoulli
(1700 – 1782)



John Larry Kelly, Jr.
(1923 – 1965)

Optimal reinvestment

The **doubling rate**:

$$W == \log R == \log \frac{\text{Capital at time } i+1}{\text{Capital at time } i}$$

So $2^W = R$.

Long-run behaviour?

Optimal reinvestment

The **doubling rate**:

$$W == \log R == \quad {}_i\, p_i \log(b_i\, o_i)$$

$$\text{So } W == \quad {}_i\, p_i \log(o_i) - \quad {}_i\, p_i \log\left(\frac{1}{b_i}\right)$$

So that…?

Optimal reinvestment

Geometric expectation

$$E[W] == \mathbf{p} \log \mathbf{bo}$$

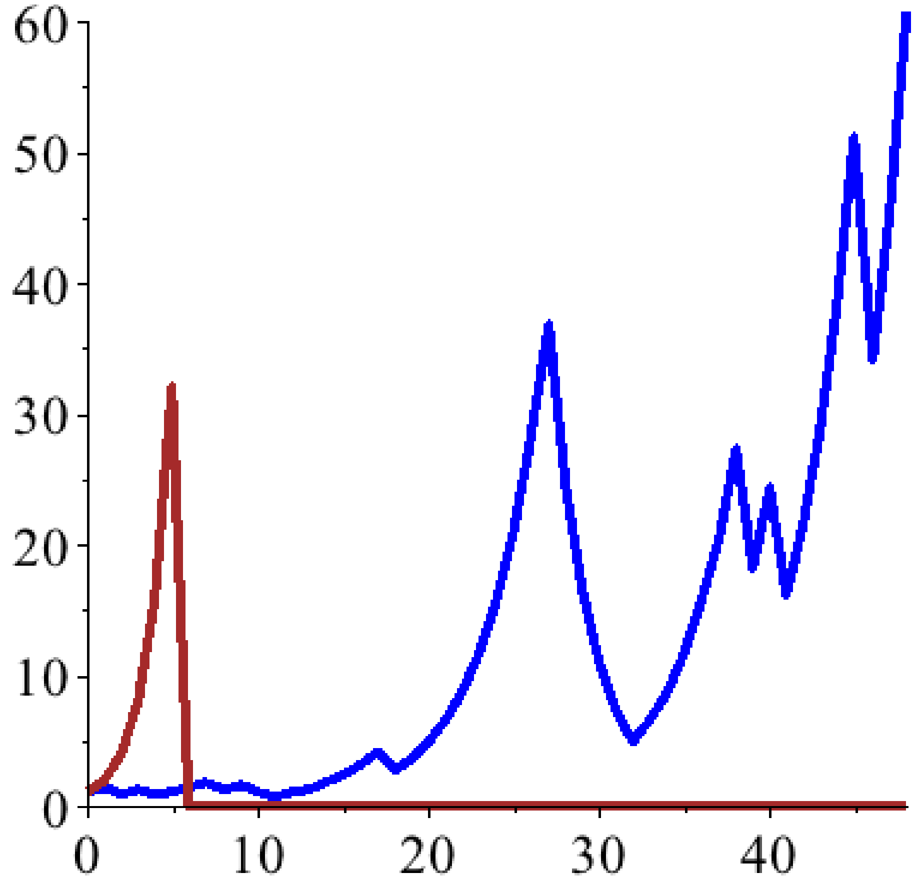
is maximized by **propor-tional gambling** (**b*** == **p**).

Arithmetic expectation

$$E[R] == \mathbf{pbo}$$

is maximized by **degenerate gambling**

Optimal reinvestment



Horse race

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Win probability	.4	.3	.2	.1
Odds	2	3	5	10

Payoffs

$$W(b) == .4 \log(2b) + .3 \log(3b) + .2 \log(5b) + .1 \log(10b)$$

Horse race

Horse number	1	2	3	4
Win probability	.4	.3	.2	.1
Odds	2	3	5	10

Payoffs

$$W(b^*) == .4\log(2\cdot .4) + .3\log(3\cdot .3) +$$
$$.2\log(5\cdot .2) + .1\log(10\cdot .1)$$

Arithmetic mean

$$\frac{1}{n}(X_1 + X_2 + X_3 + \ldots + X_n)$$

Geometric mean

$$(X_1 \cdot X_2 \cdot X_3 \cdots X_n)^{1/n}$$

For example,

$$(1/2)(2 + 8) = 5$$

$$(2 \cdot 8)^{1/2} = 4$$

The geometric mean

$$(X_1 \cdot X_2 \cdot X_3 \cdots X_n)^{1/n}$$

is also equal to

$$(2^{\log X_1 + \log X_2 + \log X_3 + \ldots + \log X_n})^{1/n}$$

which is equal to

$$(2^{1/n(\log X_1 + \log X_2 + \log X_3 + \ldots + \log X_n)})$$

so for an ergodic process X…