Exercises for Wednesday, second hour

Mathias Winther Madsen,
mathias.winther@gmail.com

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Straws (McKay, Exercise 15.4)   How can you use a fair coin to draw lots among three people? Come up with at least two different alterantives and compare them in terms of (1) fairness, and (2) expected number of coin flips.

Arithmetic coding for a bent coin   Suppose we are going to do $n = 2$ flips of a bent coin with bias $p = 1/4$.

1. Construct the arithmetic code for the outcomes of this experiment.

2. If k_i is the length of the ith codeword, what is $\sum_i 2^{-k_i}$?

3. How does that compare to the same sum for $n = 1$?

Spaced-out language   A language consists of all binary strings with no consecutive `1s`. Find a code for the set of messages from this language, assuming that the length of the message is known in advance.

Palindrome machine   A function picks an $L = 1, 2, 3, 4, \dots$ with probabilities $1/2, 1/4, 1/8, 1/16, \dots$ and then returns a binary palindrome of length L. Possible return values are, e.g., `1`, `00`, and `1001`, but not `1010` or `10`.

A machine repeatedly calls this function and prints the outputs. An output stream from this machine is thus a series of palindromes like `00 101 11 1001...`, but without the spaces.

You start this machine and observe the output

```
1100...
```

What is the probability that the next character is a `0`?

```
repeat indefinitely:

    S = ""

    while flip():

        if flip():
            print "0"
            append "0" to S

        else:
            print "1"
            append "1" to S

    print S in reverse
```