

Exercises for Thursday, second hour

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A horse race (Cover and Thomas, Exercise 6.5) A horse race has probabilities and odds as shown in the table.

Horse number	1	2	3
Probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
Odds	4	2	4

1. Find the doubling rate associated with this race.
2. Should you bet on this race?

A prediction game A person randomly selects one of the words **tip**, **top**, **tap**, **sit**, or **sip**. You can then bet on each letter of the word, one by one. Every time you guess a letter correctly, your capital is doubled.

1. What's the doubling rate associated with this game?
2. If you were the one selecting the word, and you wanted to make this game as hard as possible for the person guessing letters, what would then be your best choice as a probability distribution over these five words?

Lotto (Cover and Thomas, Exercise 6.8) In the game of Lotto, people buy lottery tickets for a fixed price and fill them in with their own "lucky numbers." A number from a fixed set is then drawn randomly, and everybody with that number gets a share of the collected money (or several shares if they hold more than one winning ticket).

People who play Lotto tend to choose some numbers more than others, for various reasons. Let's suppose that there are eight numbers available in Simplified Skewed Lotto, and that they are chosen by the following fractions of the players: $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{16}$, $\frac{1}{16}$, and $\frac{1}{16}$.

1. What's the optimal strategy for playing Simplified Skewed Lotto?
2. What doubling rate can you achieve in this game?
3. If you start out with 1 cent, how long will it take before you're a millionaire?
4. What's the highest percentage of the prize that the organizers could keep for themselves for your capital to still grow?