Exercises for Thursday, first hour

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Variational approximation Two random variables X and Y interact according to the joint probability table on the right. We will call this probability distribution P and approximate it by a distribution Q which assumes that X and Y are independent.

$P(X,Y)$	$X=1$	$X=2$
$Y=1$	0	1/2
$Y=2$	1/4	1/4

1. Which distribution over independent X and Y minimizes $D(Q\,||\,P)$?
2. Which distribution over independent X and Y minimizes $D(P\,||\,Q)$?

Competitive prediction Two scientists compete about assigning good probability estimates two the outcomes of a random process. One scientist believes that the process is a series of coin flips with bias $\theta = .6$, and the other believes that it is a series of coin flips with bias $\theta = .2$. The process is in fact a coin flipping process, but the coin actually has a bias of $\theta = .5$.

We measure the relative performance of the two scientists by looking at the likelihood ratio between their respective probability estimates,

$$\frac{\Pr(x_1, x_2, \ldots x_k \,|\, \theta = .6)}{\Pr(x_1, x_2, \ldots x_k \,|\, \theta = .2)}$$

We consider one scientists as substantially better than the other if this likelihood ratio exceeds 20 or drops below $1/20$.

Roughly how many coin flips should it take before this happens?

A substitution cipher Crack the following substitution cipher:

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GWAL VLITG IEW -- HLCLT ARHO UWF MWHE NTLBRGLMV -- UICRHE
MRDDML WT HW AWHLV RH AV NYTGL, IHO HWDURHE NITDRBYMIT DW
RHDLTLGD AL WH GUWTL, R DUWYEUD R FWYMO GIRM IPWYD I MRDDML
IHO GLL DUL FIDLTV NITD WS DUL FWTMO. RD RG I FIV R UICL WS
OTRCRHE WSS DUL GNMLLH IHO TLEYMIDRHE DUL BRTBYMIDRWH.
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Spaces and punctuation have been left unencrypted to make things easier. The underlying plaintext string is in capitalized English.