Hint for exercises for Friday, first hour

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Integer codes We can still eventually resort to repetition codes, but you should be thinking about how to compress the parameters of the compression.

In order to find the smallest integer for which the smart encoding in shorter than the dumb one, start by answering this question for an integer of the form $n = 2^r$, and then work out the finer detail later. You should get that the following pair of codewords is the smallest example in which the dumb encoding performs worse than the smart:

- Dumb: `11 00 00 00 00 00 00 00 00 01`
- Smart: `11 00 00 11 01 10 00 00 00 0`

Actually, there is a period after this (between numbers 256 and 512) in which the smart encoding is again performing worse than the dumb, but after that they truly diverge, with the smart algorithm being exponentially better than the dumb.

Line drawing For an upper bound, think about the entropy rate of the most erratic pencil movment you can imagine.

Joint and conditional Kolmogorov complexity

1. Imagine for instance that the encoding scheme is like the one we actually use in printed mathmatics: `(01110,0101101)`.
2. As examples of intuitions you should capture, consider strings like

   `0101101011010110101101011010110101101011010110101101011010110101101011`

   or conditional complexities like $K(x \mid n)$, where n is the length of x.
3. How is entropy related to information, and what properties does it have?