

# Exercises for Friday, first hour

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**Integer codes** As you have seen, a possible way of encoding a sequence is to first transmit its length in the form of a repetition code, and then transmit some identifying information about the actual message.

Instead of using a repetition code to ensure that the length information is separated from the message contents, could we use a more intelligent encoding of that initial integer? How much would that save us, and what is the smallest length were such a more complicated scheme would perform better?

**Line drawing** I draw a black and white drawing on a  $n \times n$  pixels canvas by dropping my pencil down somewhere, and then tracing a long line of length  $k$ .

Find an upper bound on the Kolmogorov complexity of such a drawing.

**Joint and conditional Kolmogorov complexity** We define  $K(x, y)$  as the length of shortest program that will print out the tuple  $(x, y)$  in some fixed encoding scheme.

1. Prove that, under reasonable assumptions about the encoding of pairs,

$$K(x, y) + \log x + c \leq K(x) + K(y).$$

2. Come up with a definition of  $K(x|y)$ .
3. What can you say about the relative sizes of  $K(x|y)$  and  $K(x)$ , and what does that tell you?